

SUPPLEMENTARY MATERIAL 1:

Proof of Interpretations of Equation 1 Coefficients

Recall equation 1, which can be rewritten:

$$y = T4 \left(e^{-\ln(2) \left(\frac{x}{H}\right)^{\frac{\ln(c)}{\ln(1+B/H)}}} - e^{-2\ln(2) \left(\frac{x}{H}\right)^{\frac{\ln(c)}{\ln(1+B/H)}}} \right) \text{ (Eq S.1)}$$

where in this case, x = plant height, y = number of twigs per plant, T = maximum number of twigs per plant, H = height at maximum number of twigs per plant, B = height increment above H at which number of twigs per plant decline to 10% of maximum, and c is a constant ($c=5.284428$), in this case written $\ln(c)$ instead of c . Using the substitution rule to take the derivative of y with respect to x gives:

$$z = \frac{\ln(c)}{\ln(1+B/H)}; u = \left(\frac{x}{H}\right)^z;$$

$$\frac{dy}{dx} = T4 \left(\frac{d}{du} (e^{-\ln(2)u}) \frac{du}{dx} - \frac{d}{du} (e^{-2\ln(2)u}) \frac{du}{dx} \right)$$

$$\frac{dy}{dx} = -\ln(2)T4 \frac{du}{dx} \left((e^{-\ln(2)u}) - (2e^{-2\ln(2)u}) \right)$$

Finding maxima and minima by setting $\frac{dy}{dx} = 0$ and solving for x gives:

$$0 = -\ln(2)T4 \frac{du}{dx} \left((e^{-\ln(2)u}) - (2e^{-2\ln(2)u}) \right)$$

$$0 = (e^{-\ln(2)u}) - (2e^{-2\ln(2)u})$$

$$2e^{-2\ln(2)u} = e^{-\ln(2)u}$$

$$\ln(2) - 2\ln(2)u = -\ln(2)u$$

$$1 - 2u = -u$$

$$1 = u$$

$$1 = \left(\frac{x}{H}\right)^{\frac{\ln(c)}{\ln(1+B/H)}}$$

$$1 = \frac{x}{H}$$

$$H = x$$

Proving the H coefficient is indeed the height at which the maximum number of twigs per plant occurs.

Substituting $x = H$ into the original equation (Eq S.1) gives:

$$y = T4 \left(e^{-\ln(2) \left(\frac{H}{H}\right)^{\frac{\ln(c)}{\ln(1+B/H)}}} - e^{-2\ln(2) \left(\frac{H}{H}\right)^{\frac{\ln(c)}{\ln(1+B/H)}}} \right)$$

$$y = T4(e^{-\ln(2)} - e^{-2\ln(2)})$$

$$y = T4 \left(\frac{1}{2} - \frac{1}{4} \right) = T$$

Proving that the T coefficient is indeed the maximum number of twigs per plant.

The height increment (B) above H at which twigs decline to 10% of maximum can be found by setting $y = T/10$ and solving for x :

$$\frac{T}{10} = T4 \left(e^{-\ln(2) \left(\frac{x}{H}\right)^{\frac{\ln(c)}{\ln(1+B/H)}}} - e^{-2\ln(2) \left(\frac{x}{H}\right)^{\frac{\ln(c)}{\ln(1+B/H)}}} \right)$$

$$\frac{1}{40} = \left(e^{-\ln(2) \left(\frac{x}{H}\right)^{\frac{\ln(c)}{\ln(1+B/H)}}} - e^{-2\ln(2) \left(\frac{x}{H}\right)^{\frac{\ln(c)}{\ln(1+B/H)}}} \right)$$

Substituting u gives:

$$\frac{1}{40} = (e^{-\ln(2)u} - e^{-2\ln(2)u}) = e^{-2\ln(2)u}(e^{\ln(2)u} - 1) = (e^{\ln(2)u})^{-2}(e^{\ln(2)u} - 1)$$

Substituting $v = e^{\ln(2)u}$ gives

$$\frac{1}{40} = \frac{v - 1}{v^2}$$

$$\frac{1}{40}v^2 - v + 1 = 0$$

Which can be solved using the quadratic equation for $a = 1/40$, $b = -1$, $c = 1$:

$$v = \frac{1 \pm \sqrt{(-1)^2 - 4 \left(\frac{1}{40}\right) (1)}}{2 * \frac{1}{40}}$$

$$v = 20 * (1 \pm \sqrt{0.9})$$

$$e^{\ln(2)u} = 20 * (1 \pm \sqrt{0.9})$$

$$u \ln(2) = \ln(20 * (1 \pm \sqrt{0.9}))$$

$$u = \frac{\ln(20) + \ln(1 \pm \sqrt{0.9})}{\ln(2)}$$

$$u = \{0.375004, 5.284428\}$$

Note that the larger number equals the constant $c=5.284428$.

Substituting back in x , z , and c gives:

$$\left(\frac{x}{H}\right)^{\frac{\ln(c)}{\ln(1+B/H)}} = c$$

$$\frac{x}{H} = c^{\frac{\ln(1+B/H)}{\ln(c)}} = e^{\ln(1+B/H)} = 1 + \frac{B}{H};$$

$$\frac{x}{H} = \frac{H+B}{H}$$

$$x = H + B$$

The meanings of parameters T , H , and B can be seen in Figure 2.

Supplementary Material 2:

Literature Derived Stem Density Parameters for Forest Management Simulation

Table S2.1. Initial stem densities and mean annual mortality rates derived from published literature for forest management simulation. Initial stem densities based on minimum initial stocking standards from listed source. Stem density at time 2 (“Stem density 2”) based on minimum stocking standards at later age from same source. Mean annual mortality estimated using $(\ln(D_1) - \ln(D_2))/Y$.

| Species | Initial Stem Density (D_1) | Stem Density 2 (D_2) | Age at Density 2 (Y) | Mean Annual Mortality | Source |
|---------------|--------------------------------|--------------------------|--------------------------|-----------------------|------------------|
| Quaking Aspen | 4,000 | 2,000 | 20 | 3.47% | Perala 1977 |
| Paper Birch | 1,900 | 385 | 40 | 3.99% | Safford 1983 |
| Red Maple | 1,430 | 385 | 59 | 2.22% | Leak et al. 1969 |

LITERATURE CITED

- LEAK, W.B., D.S. SOLOMON and S.M. FILIP. 1969. A silvicultural guide for northern hardwoods in the northeast. Upper Darby, PA: U.S. Department of Agriculture Forest Service, Northeastern Forest Experiment Station Research Paper NE-143.
- PERALA, D.A. 1977. Manager’s handbook for aspen in the north-central states. U.S. Department of Agriculture Forest Service, North Central Forest Experiment Station General Technical Report NC-36.
- SAFFORD, L.O. 1983. Silvicultural guide for paper birch in the northeast (revised). Broomall, PA: U.S. Department of Agriculture Forest Service, Northeastern Forest Experiment Station Research Paper NE-535.

Supplementary Material 3: Within Sample Model Assessment Tables

Table S3.1. Within sample RMSE, Pearson correlation coefficient, and pseudo- R^2 between observed and predicted data for unimodal and alternative models of total twig production within reach of moose at the individual plant level at Gichi Onigaming (Grand Portage Indian Reservation, Minnesota, USA) and Minong (Isle Royale, MI, USA) June-September 2019. Null model represents the mean twigs per species under open or closed canopies. Confidence intervals are at 95%.

| Model | <i>RMSE</i> | Pearson's <i>r</i> | pseudo- R^2 |
|------------|-------------|--------------------|---------------|
| Unimodal | 16.5 ± 0.24 | 0.582 ± 0.016 | 34.0% ± 1.6% |
| Allometric | 20.3 ± 0.30 | 0.355 ± 0.013 | 12.6% ± 1.3% |
| Null | 19.2 ± 0.28 | 0.326 ± 0.012 | 10.6% ± 1.2% |

Table S3.2. Within sample bias and paired t-test statistics for unimodal and alternative models of total twig production within reach of moose at the individual plant level at Gichi Onigaming (Grand Portage Indian Reservation, Minnesota, USA) and Minong (Isle Royale, MI, USA) June-September 2019. Δ Bias is the difference in bias between unimodal model and alternative models. Null model represents the mean twigs per species under open or closed canopies. Confidence intervals are at 95%.

| Model | <i>Bias</i> | <i>t</i> | <i>df</i> | <i>adj-p</i> | Δ <i>Bias</i> | <i>t</i> | <i>df</i> | <i>adj-p</i> |
|------------|--------------|----------|-----------|--------------|----------------------|----------|-----------|--------------|
| Unimodal | -0.15 ± 0.35 | -0.84 | 8517 | 1.00 | -- | -- | -- | -- |
| Allometric | 0.70 ± 0.43 | 3.18 | 8517 | 0.007 | 0.85±0.24 | 6.95 | 8517 | <0.001 |
| Null | 0.001 ± 0.41 | 0.035 | 8517 | 1.00 | 0.15±0.19 | 1.57 | 8517 | 0.43 |

Supplementary Material 4:

Out-of-sample Model Validation Tables at the Aggregated Transect Level

Table S4.1. Out-of-sample RMSE, Pearson correlation coefficient, and pseudo- R^2 between observed and predicted data for unimodal and alternative models of total twig production within reach of moose at the aggregated transect level at Gichi Onigaming (Grand Portage Indian Reservation, Minnesota, USA), September 2021. Null model represented the mean twigs per species under open or closed canopies. Confidence intervals are at 95%. Asymmetric Pseudo- R^2 confidence intervals in parentheses.

| Model | RMSE | Pearson's r | pseudo- R^2 |
|------------|----------|---------------|--------------------|
| Unimodal | 175 ± 20 | 0.739 ± 0.129 | 54.7% (41.7-65.8%) |
| Allometric | 211 ± 24 | 0.573 ± 0.136 | 32.8% (19.2-46.5%) |
| Null | 137 ± 15 | 0.540 ± 0.132 | 29.2% (15.9-43.0%) |

Table S4.2. Out-of-sample bias and paired t-test statistics for unimodal and alternative models of total twig production within reach of moose at the aggregated transect level at Gichi Onigaming (Grand Portage Indian Reservation, Minnesota, USA), September 2021. Δ Bias is the difference in bias between unimodal models and alternative models. Null model represents the mean twigs per species under open or closed canopies. Confidence intervals are 95%.

| Model | <i>Bias</i> | <i>t</i> | <i>df</i> | <i>adj-p</i> | Δ <i>Bias</i> | <i>t</i> | <i>df</i> | <i>adj-p</i> |
|------------|-------------|----------|-----------|--------------|----------------------|----------|-----------|--------------|
| Unimodal | 269 ± 31 | 16.9 | 119 | < 0.001 | -- | -- | -- | -- |
| Allometric | 298 ± 38 | 15.4 | 119 | < 0.001 | 28.6 ± 15 | 3.79 | 119 | < 0.001 |
| Null | 185 ± 25 | 14.7 | 119 | < 0.001 | -84.5 ± 25 | -6.66 | 119 | < 0.001 |

Table S4.3. Summary of observed and predicted out-of-sample validation data by species at the aggregated transect level at Gichi Onigaming (Grand Portage Indian Reservation, Minnesota, USA), September 2021. Predicted values are from the best fit unimodal model. Bold bias values are significant at $p < 0.05$.

| Species | Observed (Aggregated) | | | Predicted (Aggregated) | | | | |
|-------------------|--------------------------|-------|----------|---------------------------|----------|--------------|-------------|-------------|
| | <i>n</i> | μ | σ | μ | σ | Bias | Bias SE | <i>RMSE</i> |
| Balsam Fir | 74 | 119.6 | 117.1 | 199.3 | 223.2 | 79.7 | 27.9 | 120.2 |
| Red Maple | 3 | 17.7 | 12.6 | 55.3 | 63.0 | 37.6 | 125.8 | 50.7 |
| Sugar Maple | 30 | 26.0 | 28.4 | 81.5 | 89.8 | 55.5 | 25.0 | 67.0 |
| Mountain Maple | 84 | 58.3 | 56.1 | 162.8 | 168.2 | 104.5 | 27.3 | 125.7 |
| Speckled Alder | 29 | 25.5 | 24.0 | 49.3 | 61.9 | 23.8 | 19.7 | 51.9 |
| Juneberry Spp. | 32 | 15.4 | 16.6 | 20.2 | 24.3 | 4.7 | 3.6 | 10.1 |
| Paper Birch | 40 | 12.5 | 15.1 | 77.7 | 73.9 | 65.2 | 22.2 | 69.5 |
| Beaked Hazelnut | 87 | 37.3 | 50.0 | 94.1 | 104.6 | 56.8 | 12.8 | 60.2 |
| Red Osier Dogwood | 19 | 26.0 | 28.1 | 13.0 | 10.3 | -13.0 | 12.2 | 25.3 |
| Black Ash | 28 | 27.9 | 45.0 | 69.3 | 101.5 | 41.5 | 24.5 | 63.1 |
| White Pine | 1 | 30.0 | NA | 39.8 | NA | NA | NA | NA |
| Balsam Poplar | 5 | 12.6 | 19.3 | 23.0 | 22.6 | 10.4 | 28.7 | 23.1 |
| Quaking Aspen | 120 | 23.8 | 32.7 | 64.1 | 66.8 | 40.4 | 9.2 | 50.8 |
| Fire Cherry | 16 | 15.2 | 14.7 | 43.3 | 37.6 | 28.1 | 13.7 | 25.8 |
| Chokecherry | 13 | 12.8 | 13.3 | 69.4 | 44.0 | 56.6 | 21.9 | 36.2 |
| Willow Spp. | 11 | 45.5 | 74.2 | 83.0 | 116.9 | 37.6 | 36.5 | 54.3 |
| Red Elderberry | 4 | 70.3 | 119.9 | 47.7 | 84.7 | -22.6 | 56.1 | 35.2 |
| Mountain Ash | 24 | 10.3 | 8.8 | 15.6 | 10.2 | 5.3 | 3.7 | 8.8 |

Supplementary Material 5: Procedure and Worked Example of Application of Twig Production Equations to Forest Regeneration Field Data

We provide a brief worked example demonstrating how these equations can be applied to common forest regeneration survey data. Table S5.1 shows a typical fixed plot tally of tree and shrub regeneration by species and height class. A forester can use this tally to estimate moose forage per hectare (or acre) using the following procedure:

1. Using plot area factor, convert tally to stems per area (stems/ha or stems/ac).
2. For each species-height class combination, calculate the midpoint of the height class in meters.
3. Using the T , H , and B coefficients for that species from Table 6 and the height midpoint, use equation 1 to calculate an estimate of average twigs per stem for that species and height class. Make sure to use the appropriate open or closed canopy T coefficient for your forest condition. In this example, we use open canopy T coefficients.
4. For each species-height class combination, multiply the stems per area by the average twigs per stem from step 3. This is the estimated twigs per unit area for that species and height class.
5. For each species, sum the twigs per unit area for across all height classes. This gives the total twigs per unit area for that species.
6. For each species, look up the average biomass per twig estimate from Table 3 for summer or Table 2 for winter in Portinga and Moen (2015) or other references. In this example, we used summer values.
7. For each species, multiply the estimated twigs per area from step 5 by the mass per twig in step 6. This is the forage density available to moose for that species in grams per area. Convert to kg per area by dividing by 1000 or to lbs per area by dividing by 453.592.

8. For each species, look up moose preference classes from Table 8 in Portinga and Moen (2015) or other references. Moose preferences can be “preferred”, “neutral use” (used according to availability), or “avoided” (rarely used). In this example, we used summer preference classes.

9. For each preference class, sum the biomass per unit area across all species in that preference class from step 7. This is the total forage density available to moose by preference.

In this example, we estimate that in summer, there are 61 kg/ha of preferred forage, 486 kg/ha of neutral forage (used proportional to availability), and 49 kg/ha of forage avoided by moose (Table E.1). Winter estimates would differ with changes in preference and mass per twig.

Because Equation 1 is non-linear, interpolation methods should yield more accurate twigs per stem estimates. In this example, total twig density was 1.0% lower using the simple average interpolation method (i.e. averaging twig per stem estimates of the low bound, midpoint, and upper bound of each height class).

LITERATURE CITED

PORTINGA, R.L.W. and R.A. MOEN. 2015. A novel method of performing moose browse surveys. *Alces* 51:107–122.

Table S5.1. Worked example of estimating moose forage density from forest regeneration survey data by species and height class. Twigs per stem are estimated from EQ. 1 [$y = 4T(\exp(-\ln(2)(x/H)^{(c/\ln(1+B/H))}) - \exp(-2\ln(2)(x/H)^{(c/\ln(1+B/H))}))$] where y is twigs per plant, x is plant height in meters, $c = \ln(5.284428)$, and coefficients T , H , B , are from Table 6 according to species and canopy cover. Mass per twig estimates from Table 3 and moose preferences from Table 8 of Portinga and Moen (2015). In this example, we assume a 1/250 acre plot size (7.4 ft radius), an open canopy, and summer season for twig biomass and preference.

| Species | Height Class | Tally | Stems Per Acre | Ht. Class M.P. (ft) | Ht. Class M.P. (m) | T | H | B | Twigs per Stem | Twig Density (#/ac) | Mass per Twig (g) | Forage Dens (kg/ha) | Preference |
|-------------------------|--------------|-------|----------------|---------------------|--------------------|------|-----|------|----------------|---------------------|-------------------|---------------------|------------|
| Mountain Maple | 2-4 ft | 13 | 3,250 | 3 | 0.9 | 25.0 | 3.4 | 8.0 | 9.6 | 31,191 | 0.7 | 54 | Preferred |
| Mountain Maple | 4-6 ft | 1 | 250 | 5 | 1.5 | 25.0 | 3.4 | 8.0 | 16.3 | 4,079 | 0.7 | 7 | Preferred |
| M. Maple Subtotal | | | 3,500 | | | | | | | 35,269 | | 61 | |
| Total Preferred Spp. | | | 3,500 | | | | | | | 35,269 | | 61 | |
| Quaking Aspen | 2-4 ft | 12 | 3,000 | 3 | 0.9 | 23.9 | 2.4 | 4.5 | 11.5 | 34,637 | 1.4 | 120 | Neutral |
| Quaking Aspen | 4-6 ft | 4 | 1,000 | 5 | 1.5 | 23.9 | 2.4 | 4.5 | 19.6 | 19,577 | 1.4 | 68 | Neutral |
| Quaking Aspen | 6-8 ft | 5 | 1,250 | 7 | 2.1 | 23.9 | 2.4 | 4.5 | 23.5 | 29,412 | 1.4 | 102 | Neutral |
| Quaking Aspen | 8-12 ft | 2 | 500 | 10 | 3.0 | 23.9 | 2.4 | 4.5 | 22.1 | 11,067 | 1.4 | 38 | Neutral |
| Q. Aspen Subtotal | | | 5,750 | | | | | | | 94,694 | | 328 | |
| Paper Birch | 2-4 ft | 3 | 750 | 3 | 0.9 | 54.1 | 6.3 | 19 | 13.4 | 10,063 | 0.8 | 20 | Neutral |
| Paper Birch | 4-6 ft | 6 | 1,500 | 5 | 1.5 | 54.1 | 6.3 | 19 | 22.7 | 34,031 | 0.8 | 67 | Neutral |
| Paper Birch | 6-8 ft | 2 | 500 | 7 | 2.1 | 54.1 | 6.3 | 19 | 30.9 | 15,457 | 0.8 | 31 | Neutral |
| Paper Birch | 8-12 ft | 2 | 500 | 10 | 3.0 | 54.1 | 6.3 | 19 | 40.8 | 20,404 | 0.8 | 40 | Neutral |
| P. Birch Subtotal | | | 3,250 | | | | | | | 79,955 | | 158 | |
| Total Neutral Spp. | | | 9,000 | | | | | | | 174,649 | | 486 | |
| Beaked Hazelnut | 2-4 ft | 1 | 250 | 3 | 0.9 | 40 | 2.4 | 6 | 23.1 | 5,773 | 0.7 | 10 | Avoided |
| B. Hazelnut Subtotal | | | 250 | | | | | | | 5,773 | | 10 | |
| Balsam Fir | 2-4 ft | 1 | 250 | 3 | 0.9 | 48.3 | 4.7 | 12.5 | 14.4 | 3,609 | 1.6 | 14 | Avoided |
| Balsam Fir | 4-6 ft | 1 | 250 | 5 | 1.5 | 48.3 | 4.7 | 12.5 | 24.7 | 6,183 | 1.6 | 24 | Avoided |
| Balsam Fir Subtotal | | | 500 | | | | | | | 9,791 | | 38 | |
| Total Avoided Spp. | | | 750 | | | | | | | 15,564 | | 49 | |
| Total Forage (All Spp.) | | | 13,250 | | | | | | | 225,483 | | 595 | |